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Type -B/ -O Bosonic String Sigma-Models

S. James Gates, Jr.¹

*Department of Physics
University of Maryland at College Park
College Park, MD 20742-4111, USA*

and

V.G.J. Rodgers²
*Department of Physics and Astronomy
University of Iowa
Iowa City, Iowa 52242-1479*

ABSTRACT

We provide world sheet non-supersymmetrical actions that describe the coupling of a bosonic string to the tachyon and massless states of both the type-B and type-O theories. The type-B theory is derived as a truncation and chiral doubling of the Ramond-Ramond sector in our previous model that connected the (1,0) heterotic string to a 10D, type-IIB supergravity background. The type-O theory then follows from a “fermionization” of the type-B theory.

¹gates@umdhep.umd.edu

²vincent@hepaxp.physics.uiowa.edu

1 Introduction

Approximately two years ago, we pointed out a curious feature [1] of (1,0) supergravity NSR non-linear σ -models and type-IIB supergravity. Although totally unexpected at the time, we formulated a coupling of a heterotic string [2] to a background that consisted of the massless bosonic fields of the type-IIB supergravity theory. The reason for this to be unexpected was that it had always been thought that the type-IIB supergravity theory (whose superspace description is well known [3]) was only associated with the type-IIB superstring of Green and Schwarz [4]. From this point of view it was very “unnatural” to find a heterotic model associated with the massless bosonic fields of type-IIB supergravity.

On the other hand, the construction of a (1,0) NSR non-linear σ -model coupled to a type-IIB supergravity background cleared up an earlier puzzle. Several years prior to the work in [1], it was noted that the (1,0) heterotic string nonlinear σ -model possessed exactly the right properties to allow a coupling to a 4D, $N = 8$ supergravity background also [5]. This 4D result can now be viewed as the simple dimensional reduction of the type-IIB heterotic string nonlinear σ -model.

In order to accomplish the result in [1], we introduced a feature which had not appeared in the context of stringy non-linear σ -models prior to that time. The idea was that when certain combinations of p-forms occur in the Ramond-Ramond sector of a superstring theory, they correspond to the introduction of WZNW models on the world sheet of the superstring where the currents of the WZNW model do not represent internal symmetries but instead are associated with the Clifford algebra of the target spacetime. This concept has reappeared recently in discussions of Dirichlet p-branes and type-IIB supergravity [6].

Since the model of [1] is a (1,0) theory, when analyzed in terms of its left and right handed degrees of freedom, one finds the expected result that the left-handed sector is supersymmetric while the right-handed sector is non-supersymmetric. This observation is the key to the construction of the non-supersymmetric nonlinear σ -model for the type-B string. In the language of 2D field theory we well know how to take an ordinary boson and separate it into its left-handed and right-handed segments by use of chiral bosons [7]. Once this is done, the original left-handed supersymmetric sector may be discarded. However, in order to have a consistent closed bosonic string, it is required to have some non-supersymmetric left-handed sector. A new left-handed sector may be introduced by constructing the “mirror” of the right-handed sector of the original (1,0) type-IIB heterotic string non-linear σ -model.

There are two steps required for constructing the mirror. First, all the 2D fields that depended only the σ^- -coordinate on-shell must be replaced by fields that depend

on σ^\sharp -coordinate on-shell³. Also the target spacetime Clifford algebra of the original Ramond-Ramond sector of the (1,0) model must be replaced by its target spacetime chiral reflection⁴. This mirror is the ‘glued’ back to the original right handed sector and the result is the type-B nonlinear σ -model with precisely the couplings to the massless spectrum of the type-B string.

2 Review of NSR Supergravity-Heterotic Sigma-Models in (1,0) Superspace

Although the original formulation of the heterotic string [2] was *not* as a (1,0) superfield theory, with the work of [8], it became possible to show that the action given by

$$S_{HET-1} = \frac{1}{4\pi\alpha'} \int d^2\sigma d\zeta^- E^{-1} \left\{ i\eta_{\underline{mn}} (\nabla_+ X^{\underline{m}}) (\nabla_- X^{\underline{n}}) - (\eta_-^{\hat{I}} \nabla_+ \eta_-^{\hat{I}}) \right\} , \quad (2.1)$$

upon gauge-fixing to eliminate the (1,0) supergravity fields yields the action of Gross et. al. in the fermionic formulation (i.e. $\hat{I} = 1, \dots, 32$). A second formulation (also given by Gross et. al.) utilizing chiral bosons [7] whose (1,0) superspace form is

$$S_{HET-2} = \int d^2\sigma d\zeta^- E^{-1} \left\{ \frac{1}{4\pi\alpha'} [i\eta_{\underline{mn}} (\nabla_+ X^{\underline{m}}) (\nabla_- X^{\underline{n}})] + \right. \\ \left. i\frac{1}{2} [(\nabla_+ \Phi_R^{\hat{a}}) (\nabla_- \Phi_R^{\hat{a}}) + \Lambda_-^\sharp (\nabla_+ \Phi_R^{\hat{a}}) (\nabla_\sharp \Phi_R^{\hat{a}})] \right\} , \quad (2.2)$$

where $\hat{a} = 1, \dots, 16$ was also described. Neither of these two formulations allows for a manifest realization of the $E_8 \otimes E_8$ symmetry of heterotic string theory. To accomplish this, it is necessary to carry out a non-abelian bosonization [9] of the fermionic superfields $(\eta_-^{\hat{I}})$ in the first action above⁵.

$$S_{HET-3} = \frac{1}{4\pi\alpha'} \int d^2\sigma d\zeta^- E^{-1} [i\eta_{\underline{mn}} (\nabla_+ X^{\underline{m}}) (\nabla_- X^{\underline{n}})] \\ - \frac{1}{2\pi} \int d^2\sigma d\zeta^- E^{-1} i\frac{1}{2} Tr \{ R_+ R_- + i\Lambda_-^\sharp R_+ \nabla_+ R_+ \\ + \frac{2}{3} \Lambda_-^\sharp \{ R_+, R_+ \} R_+ \\ + \int_0^1 dy [(\frac{d\tilde{U}}{dy} \tilde{U}^{-1}) [\nabla_- ((\nabla_+ \tilde{U}) \tilde{U}^{-1}) - \nabla_+ ((\nabla_- \tilde{U}) \tilde{U}^{-1})]] \} . \quad (2.3)$$

³In the language of conformal field theory this corresponds to $X(z) \rightarrow \tilde{X}(\bar{z})$.

⁴All the undotted 10D spinor indices must be replaced by dotted 10D spinor indices.

⁵The action in (2.2) may be regarded as an abelian bozonization of (2.1).

where the following definitions are used,

$$R_a \equiv U^{-1} \nabla_a U \quad , \quad U \equiv \exp \left[i \Phi_R^{\hat{a}} t_{\hat{a}} \right] \quad . \quad (2.4)$$

In this last equation, the quantities $\Phi_R^{\hat{a}}$ constitute 496 righton (1,0) superfields containing the same number of component rightons. From this last equation it is clear that in order to give a complete specification of the action in (2.3), it is necessary to introduce a set of matrices (above denoted by $t_{\hat{a}}$) that form a Lie algebra. In fact, the action of (2.3) permits us to associate a *distinct* world sheet action with *each* consistent 10D heterotic string. It has long been known [10] that the groups that lead to tachyon-free 10D heterotic strings are; $E_8 \otimes E_8$, $SO(32)$ and $SO(16) \otimes SO(16)$. Thus the matrices $t_{\hat{a}}$ may be chosen to provide representations of these algebras and there is no need to introduce winding modes to construct the $E_8 \otimes E_8$ representation.

No background fields at all appear in the actions above. The way to introduce background spacetime fields is of course well known [11]. The NS-NS bosonic fields (g_{mn} , b_{mn} and Φ) are made to appear by replacing the first line of (2.3) by

$$\begin{aligned} S_{NS} &= \frac{1}{2\pi\alpha'} \int d^2\sigma d\zeta^- E^{-1} [i\frac{1}{2}(g_{mn}(X) + b_{mn}(X)) (\nabla_+ X^m)(\nabla_- X^n)] \\ &\quad + \int d^2\sigma d\zeta^- E^{-1} \Phi(X) \Sigma^+ \\ &\equiv \int d^2\sigma d\zeta^- E^{-1} [i\frac{1}{2}(\eta_{ab} + b_{ab}(X)) \Pi_+^a \Pi_-^b + \Phi(X) \Sigma^+] \quad , \end{aligned} \quad (2.5)$$

where $\Pi_+^a \equiv (1/\sqrt{2\pi\alpha'}) (\nabla_+ X^m) e_{\underline{m}}^a$ and $\Pi_-^a \equiv (1/\sqrt{2\pi\alpha'}) (\nabla_- X^m) e_{\underline{m}}^a$.

The introduction of the R-R bosonic fields (i.e. the spacetime gauge fields $A_{\underline{a}\hat{i}\hat{j}}$) for the internal symmetry groups can be carried out for the $SO(32)$ and $SO(16) \otimes SO(16)$ theories quite easily [12]. The last term in (2.1) is replaced by,

$$S_R = -\frac{1}{2} \int d^2\sigma d\zeta^- E^{-1} [\eta_-^{\hat{i}} \nabla_+ \eta_-^{\hat{i}} + \Pi_+^a \eta_-^{\hat{i}} A_{\underline{a}\hat{i}\hat{j}}(X) \eta_-^{\hat{j}}] \quad . \quad (2.6)$$

However, the introduction of the R-R bosonic fields in the $E_8 \otimes E_8$ theory can *only* be done by modifying the last three terms in (2.3). These must be replaced by [13],

$$\begin{aligned} S'_R &= -\frac{1}{2\pi} \int d^2\sigma d\zeta^- E^{-1} i\frac{1}{2} Tr \{ (R_+ + 2\Gamma_+) R_- \\ &\quad + i\Lambda_-^{\sharp} (R_+ + \Gamma_+) \nabla_+ (R_+ + \Gamma_+) \\ &\quad + \frac{2}{3} \Lambda_-^{\sharp} \{ (R_+ + \Gamma_+) , (R_+ + \Gamma_+) \} (R_+ - \frac{1}{2} \Gamma_+) \\ &\quad + \int_0^1 dy [(\frac{d\tilde{U}}{dy} \tilde{U}^{-1}) [\nabla_- ((\nabla_+ \tilde{U}) \tilde{U}^{-1}) - \nabla_+ ((\nabla_- \tilde{U}) \tilde{U}^{-1})]] \} \quad . \end{aligned} \quad (2.7)$$

where $\Gamma_+ \equiv \Pi_+{}^a A_a{}^{\hat{a}}(X) t_{\hat{a}}$. Since the non-abelian bosonized theory offers the most complete description, we will only utilize it in the subsequent discussion.

In addition to these three 10D tachyon-free heterotic string theories, there are also a number of other non-supersymmetric theories that contain a tachyon [10]. These include $SO(32)$, $E_8 \otimes SO(16)$, $SO(24) \otimes SO(8)$, $(E_7 \otimes SU(2))^2$, $SU(16) \otimes U(1)$, E_8 . Although it appears not generally known, the (1,0) supergeometry also permits the introduction of a coupling of the tachyon to the worldsheet of the heterotic string. This is accomplished by a slight generalization of a result presented some time ago [8]. The construction requires the introduction of a world sheet minus spinor superfield Ψ_- that appears in the action

$$S_{tachyon} = \int d^2\sigma d\zeta^- E^{-1} \left[-\frac{1}{2}(\Psi_- \nabla_+ \Psi_-) + iT(X)\Psi_- \right] . \quad (2.8)$$

In the limit where $T(X) = 1$, this action introduces a cosmological constant on the worldsheet. Note that each of the non-supersymmetric models corresponds to a distinct choice of the matrix generators $t_{\hat{a}}$ for the groups listed above.

Up until our work of [1], as far as was known, the models above described all 10D (1,0) supergravity NSR heterotic σ -models. With this work we proposed the addition of one more such model. The introduction of this final model, begins with the observation that the $E_8 \otimes E_8$ algebra in (2.4) may be replaced by another (non-compact) Lie algebra so that the new group elements take the form

$$U \rightarrow U_{R-1} \equiv \exp \left[\Phi_R \delta_\alpha{}^\beta + \frac{1}{2} \Phi_{\underline{ab}R} (\sigma^{\underline{ab}})_\alpha{}^\beta + \frac{1}{24} \Phi_{\underline{abcd}R} (\sigma^{\underline{abcd}})_\alpha{}^\beta \right] \quad (2.9)$$

where the 10D Pauli matrices above correspond to the right-handed spacetime chiral projection of the 10D gamma matrices! This structure introduces 256 (1,0) righton superfields (Φ_R , $\Phi_{\underline{ab}R}$ and $\Phi_{\underline{abcd}R}$) instead of the familiar 496 (1,0) righton superfields of the $E_8 \otimes E_8$ model. The σ -model associated with this alternative construction is obtained by using the group elements in (2.9) to replace those in (2.4). As well the quantity Γ_+ below (2.7) must be replaced by

$$\Gamma_+ \rightarrow \Pi_+{}^a \left[(\nabla_a A) \delta_\alpha{}^\beta + \frac{1}{2} F_{\underline{abc}} (\sigma^{\underline{bc}})_\alpha{}^\beta + \frac{1}{24} F^{(+)}_{\underline{abcde}} (\sigma^{\underline{bcde}})_\alpha{}^\beta \right] \quad (2.10)$$

The R-R bosonic fields here are precisely what is needed so that their addition to the NS-NS fields describes the bosonic spectrum of 10D, type-IIB supergravity. The σ -model we have described can easily be studied upon performing dimensional compactification. These actions provide an intrinsic description of the models that were recently studied by Maharana [14].

Thus, we argued that among the family of 10D heterotic string theories, by different choices of the matrix generators we could describe all previous known models

in a uniform manner. As we found there seemed to exist one more member of this family that corresponds to replacing the internal compact group generators by the non-compact chiral 10D Pauli matrices and gives rise to a heterotic string model with a $N = 2B$ supersymmetry.

Finally there is an intriguing interpretation that we can give to (2.3) as modified in (2.9) and (2.10). Within the context of superfield theories, it is well known that it is possible to use low N superfields to realize a theory with a higher than N supersymmetry. Some examples of this are the use of 4D, $N = 1$ superfield perturbation theory to describe 4D, $N = 4$ supersymmetric Yang-Mills theory [15] or to describe 4D, $N = 2$ supersymmetric Yang-Mills theory [16]. In the same way, our work suggests that it is possible to realize the $N = 2B$ superstring as a theory of $N = 1$ superstrings coupled to a certain “matter superstring” containing the R-R sector p-forms.

3 10D Green-Schwarz Heterotic Sigma-Models

In the last section, we used (1,0) superfields to review the situation of (1,0) supergravity heterotic σ -models. Since it is our eventual goal to show that the last model above is very closely related to the type-B and type-O bosonic string theories, as a first step we begin to eliminate the (1,0) world sheet supersymmetry by going to a Green-Schwarz type formulation. The relevant starting points are (2.5) and (2.7). The action in (2.5) is replaced by \mathcal{S}_{GS}

$$S_{GS} = \int d^2\sigma V^{-1} \left[-\frac{1}{2} \Pi_{\mp}^{\underline{a}} \Pi_{\pm \underline{a}} + \int_0^1 dy \hat{\Pi}_y^{\underline{C}} \hat{\Pi}_{\mp}^{\underline{B}} \hat{\Pi}_{\pm}^{\underline{A}} \hat{G}_{\underline{ABC}} + \Phi(Z) \mathcal{R}(V) \right] ,$$

$$\Pi_{\mp}^{\underline{A}} = V_{\mp}^{\ m} (\partial_m Z^{\underline{M}}) E_{\underline{M}}^{\underline{A}} , \quad \Pi_{\pm}^{\underline{A}} = V_{\pm}^{\ m} (\partial_m Z^{\underline{M}}) E_{\underline{M}}^{\underline{A}} ,$$

$$\hat{Z}^{\underline{M}} = Z^{\underline{M}}(\sigma, \tau, y) , \quad \hat{\Pi}_y^{\underline{A}} = (\partial_y \hat{Z}^{\underline{M}}) E_{\underline{M}}^{\underline{A}}(\hat{Z}) , \quad \hat{G}_{\underline{ABC}} = G_{\underline{ABC}}(\hat{Z}) . \quad (3.1)$$

here $Z^{\underline{M}}(\tau, \sigma)$ is the superstring coordinate ($Z^{\underline{M}}(\tau, \sigma) \equiv (\Theta^{\mu}(\tau, \sigma), X^{\underline{m}}(\tau, \sigma))$), $G_{\underline{ABC}}$ is the field strength supertensor for a super 2-form $b_{\underline{AB}}(Z)$, $\Phi(Z)$ is the spacetime dilaton and $\mathcal{R}(V)$ is the world sheet curvature tensor. Since only in the cases of the $E_8 \otimes E_8$ or $SO(32)$ theories is there spacetime supersymmetry⁶, we do not at this stage have to worry about the introduction of a term to accommodate the tachyon that occurs for the other heterotic strings.

The replacement action for (2.7) can be obtained in the following manner. The action in (2.7) is a (1,0) superfield action, which using standard techniques, we have many times previously analyzed in terms of its component fields. The component

⁶The only reason to use a Green-Schwarz formulation is precisely the presence of spacetime supersymmetry.

field formulation contains fermions (none of which are dynamical), so to pass to an action that is compatible with the Green-Schwarz type action above in (3.1), we simply set all fermions to zero. This leaves the action

$$\begin{aligned}
S_R = & -\frac{1}{4\pi} \int d^2\sigma V^{-1} \text{Tr} \{ (\mathcal{D}_\mp U^{-1})(\mathcal{D}_\pm U) + \lambda_\pm^\mp (U^{-1} D_\mp U)^2 \\
& + \int_0^1 dy (\tilde{U}^{-1} \frac{d}{dy} \tilde{U}) [(\mathcal{D}_\mp \tilde{U}^{-1})(\mathcal{D}_\pm \tilde{U}) - (\mathcal{D}_\pm \tilde{U}^{-1})(\mathcal{D}_\mp \tilde{U})] \\
& - 2 \Pi_\mp \underline{B} \Gamma_{\underline{B}}^{\hat{a}} t_{\hat{a}} (U^{-1} \mathcal{D}_\pm U) \} \quad , \quad (3.2)
\end{aligned}$$

with $D_\mp U \equiv \mathcal{D}_\mp U - i \Pi_\mp \underline{B} \Gamma_{\underline{B}}^{\hat{a}} U t_{\hat{a}}$. The quantity $U \equiv \exp[i \phi_R^{\hat{a}}(\tau, \sigma) t_{\hat{a}}]$ is an element of an *a priori* arbitrary group. The matrices $t_{\hat{a}}$ generate a compact Lie algebra for the right-gauge group \mathcal{G}_R where $\hat{a} = 1, \dots, d_G$, $[t_{\hat{a}}, t_{\hat{b}}] = i f_{\hat{a}\hat{b}}^{\hat{c}} t_{\hat{c}}$, $f_{\hat{a}\hat{b}\hat{c}} f^{\hat{a}\hat{b}}_{\hat{d}} = c_2 \delta_{\hat{c}\hat{d}}$, and $\text{Tr}(t_{\hat{a}} t_{\hat{b}}) = 2k \delta_{\hat{a}\hat{b}}$. Above, we have used the notation $(\mathcal{D}_\mp, \mathcal{D}_\pm)$ to denote the world-sheet two-dimensional gravitationally covariant derivative. To describe the two tachyon-free 10D theories, we pick $\mathcal{G}_R = E_8 \otimes E_8$ or $SO(32)$, respectively.

At this stage, we once again replace the compact group generators and their corresponding 2D righton fields as described in (2.9) but here all the superfields (the Φ 's) are replaced by a 2D Duffin-Kemmer-Petiau field ϕ_α^β

$$\phi_\alpha^\beta = \phi_R \delta_\alpha^\beta + \frac{1}{2} \phi_{\underline{ab}R} (\sigma^{\underline{ab}})_\alpha^\beta + \frac{1}{24} \phi_{\underline{abcd}R} (\sigma^{\underline{abcd}})_\alpha^\beta \quad . \quad (3.3)$$

One final step is that the spacetime gauge superconnection and generators on the last line of (3.2) must undergo the replacement

$$\Gamma_{\underline{A}}^{\hat{a}} t_{\hat{a}} \rightarrow \left[(\nabla_{\underline{A}} \mathbf{A}) \delta_\alpha^\beta + \frac{1}{2} F_{\underline{Abc}} (\sigma^{\underline{bc}})_\alpha^\beta + \frac{1}{24} F^{(+)}_{\underline{Abcde}} (\sigma^{\underline{bcde}})_\alpha^\beta \right] \quad . \quad (3.4)$$

Thus, we recover (within the context of a Green-Schwarz formulation) the exact same result as seen from the NSR formulation. Namely, a heterotic 10D, $N = 1$ Green-Schwarz σ -model with manifest $E_8 \otimes E_8$ symmetry is replaced by a model where the spectrum of field strength superfields in the R-R sector is exactly what one needs to describe the type-IIB theory.

4 On to Bosonic Type-B and Type-O Theories

In this final section we complete the journey from the heterotic type-IIB σ -model to the purely non-supersymmetric type-B and type-O models. This is done in several stages. First since these final models are bosonic, we can begin with (3.1) and simply set the Grassmann superstring coordinates (Θ) identically to zero. Since

we want to allow coupling to the tachyon, we also add one additional term to find the NS-NS sector of the type-B theory takes the form

$$\begin{aligned}
S_{NS}^B &= \frac{1}{2\pi\alpha'} \int d^2\sigma V^{-1} \left[\frac{1}{2} (g_{\underline{mn}}(X) + b_{\underline{mn}}(X)) (\nabla_{\mp} X^{\underline{m}}) (\nabla_{\pm} X^{\underline{n}}) \right] \\
&\quad + \int d^2\sigma V^{-1} [\Phi(X) \mathcal{R} + T(X)] \\
&\equiv \int d^2\sigma V^{-1} \left[\frac{1}{2} (\eta_{\underline{ab}} + b_{\underline{ab}}(X)) \Pi_{\mp}^{\underline{a}} \Pi_{\pm}^{\underline{b}} + \Phi(X) \mathcal{R} + T(X) \right] .
\end{aligned} \tag{4.1}$$

This looks almost exactly like (2.5). There is one important difference of course. All of the quantities in (2.5) are (1,0) superfields whereas all quantities appearing here are ordinary 2D fields.

For the righton R-R sector of the type-B theory, we take the action of (3.2) and simply set to zero the Grassmann superstring coordinates (Θ) to zero.

$$\begin{aligned}
S_{R-1}^B &= -\frac{1}{4\pi} \int d^2\sigma V^{-1} Tr \{ (\mathcal{D}_{\mp} U_R^{-1}) (\mathcal{D}_{\pm} U_R) + \lambda_{\pm}^{\mp} (U_R^{-1} \mathcal{D}_{\mp} U_R)^2 \\
&\quad + \int_0^1 dy (\tilde{U}_R^{-1} \frac{d}{dy} \tilde{U}_R) [(\mathcal{D}_{\mp} \tilde{U}_R^{-1}) (\mathcal{D}_{\pm} \tilde{U}_R) - (\mathcal{D}_{\pm} \tilde{U}_R^{-1}) (\mathcal{D}_{\mp} \tilde{U}_R)] \\
&\quad - 2 \Pi_{\mp}^{\underline{a}} [(\nabla_{\underline{a}} A^1) + \frac{1}{2} F_{\underline{abc}}^1 (\sigma^{\underline{bc}}) + \frac{1}{24} F^{(+)}_{\underline{abcde}} (\sigma^{\underline{bcde}})] (U_R^{-1} \mathcal{D}_{\pm} U_R) \} ,
\end{aligned} \tag{4.2}$$

The actions in (4.1) and (4.2) cannot comprise the entirety of the type-B theory. We obtained the latter of these directly from the GS action of the previous section. To complete the type-B theory we must construct the “mirror” to (4.2) above. In the mirror action all indices of the \mp type are exchanged for the \pm type and vice-versa. However, this is not sufficient. In addition we must replace the non-supersymmetric limit of U_{R-1} by its mirror also. This is done as follows.

In 10D, the notion of chiral spinors exists. In fact, the matrices that appear in (2.9) are in such a basis. There also exist 10D matrices where we exchange the handedness of the spacetime spinor indices (i.e. $\alpha \rightarrow \dot{\alpha}$ etc.). Thus we may introduce U_L according to the definition,

$$U_L \equiv \exp \left[\phi_L \delta_{\dot{\alpha}}^{\dot{\beta}} + \frac{1}{2} \phi_{\underline{ab}L} (\sigma^{\underline{ab}})_{\dot{\alpha}}^{\dot{\beta}} + \frac{1}{24} \phi_{\underline{abcd}L} (\sigma^{\underline{abcd}})_{\dot{\alpha}}^{\dot{\beta}} \right] \tag{4.3}$$

which introduces 256 lefton fields $(\phi_L, \phi_{\underline{ab}L}$ and $\phi_{\underline{abcd}L})$. Like its right handed mirror counterpart, the argument of the exponential function above can be shown to form a Lie algebra. In fact, its composition law is exactly the same as that given for the right handed one in ref. [1], with the exception that the term involving the Levi-Civita tensor has the opposite sign. The complete “mirror” action to (4.2) is

given by

$$\begin{aligned}
S_{R-2}^B = & -\frac{1}{4\pi} \int d^2\sigma V^{-1} \text{Tr} \{ (\mathcal{D}_\# U_L^{-1})(\mathcal{D}_= U_L) + \lambda_\#^=(U_L^{-1} D_= U_L)^2 \\
& - \int_0^1 dy (\tilde{U}_L^{-1} \frac{d}{dy} \tilde{U}_L) [(\mathcal{D}_\# \tilde{U}_L^{-1})(\mathcal{D}_= \tilde{U}_L) - (\mathcal{D}_= \tilde{U}_L^{-1})(\mathcal{D}_\# \tilde{U}_L)] \\
& - 2 \Pi_\#^a [(\nabla_a A^2) + \frac{1}{2} F_{\underline{abc}}^2 (\bar{\sigma}^{\underline{bc}}) + \frac{1}{24} F^{(-)}_{\underline{abcde}} (\bar{\sigma}^{\underline{bcde}})] (U_L^{-1} \mathcal{D}_= U_L) \} .
\end{aligned} \tag{4.4}$$

On the final line above, we have placed bars above the 10D Pauli matrices to indicate that these are the ones with the dotted spacetime spinor indices. So the complete non-linear σ -model for the type-B theory is just

$$S_{Tot}^B = S_{NS}^B + S_{R-1}^B + S_{R-2}^B . \tag{4.5}$$

It can be seen that the complete spectrum of spacetime fields includes the graviton (g_{mn}), axion (b_{mn}), dilaton (Φ), tachyon (T), two scalars (A^1 and A^2)⁷, two Kalb-Ramond fields (A_{mn}^1 and A_{mn}^2), 4-form $A_{\underline{abcd}}^+$ with a self-dual field strength $F^{(+)}_{\underline{abcde}}$ and a second 4-form $A_{\underline{abcd}}^-$ with an anti-self-dual field strength $F^{(-)}_{\underline{abcde}}$.

The key to our successful construction of the model described immediately above is the suggestion that currents associated with internal symmetries can sometimes be traded for currents associated with the Clifford algebra of the spacetime spinors. We first did this in the 10D supersymmetric theory. If this conjecture is accepted, then the reverse is likely to also be true. With this as a working assumption, we will now show that the type-O string [18] propagating in the presence of its tachyon and massless modes can be constructed from the corresponding type-B theory.

So we once more will trade some of the currents. In particular we note that by keeping only the middle terms in the exponential of (3.3) and (4.3) we also maintain the structure of a Lie group. So the simple idea is to trade the currents associated with the 0-forms and 4-forms and replace them by currents associated with $SO(32)$. This is all easily done for the σ -models and leads to the following type-O action. The final answer is rather tedious to write only because it possesses a number of different R-R sectors but takes the form

$$S_{Tot}^O \equiv S_{NS}^O + S_{R-1}^O + S_{R-2}^O + S_{R-3}^O + S_{R-4}^O . \tag{4.6}$$

Below we give each sub-action.

$$S_{NS}^O = \int d^2\sigma V^{-1} \left[\frac{1}{2} \eta_{\underline{ab}} \Pi_\#^{\underline{a}} \Pi_\#^{\underline{b}} + \Phi(X) \mathcal{R} + T(X) \right] . \tag{4.7}$$

⁷An interesting feature to note is that the scalars A^1 and A^2 possess “moduli” (i.e. we may shift them by constants without affecting the action) unlike the scalars that usually appear in Lefton-Righton Thirring Models [13].

$$\begin{aligned}
S_{R-1}^O = & -\frac{1}{4\pi} \int d^2\sigma V^{-1} \text{Tr}\{ (\mathcal{D}_\# U_R^{-1})(\mathcal{D}_= U_R) + \lambda_=^\# (U_R^{-1} D_\# U_R)^2 \\
& + \int_0^1 dy (\tilde{U}_R^{-1} \frac{d}{dy} \tilde{U}_R) [(\mathcal{D}_\# \tilde{U}_R^{-1})(\mathcal{D}_= \tilde{U}_R) - (\mathcal{D}_= \tilde{U}_R^{-1})(\mathcal{D}_\# \tilde{U}_R)] \\
& - \Pi_\# \stackrel{a}{=} [F_{\underline{abc}}^1 (\sigma^{\underline{bc}})] (U_R^{-1} \mathcal{D}_= U_R) \} \quad , \quad (4.8)
\end{aligned}$$

$$\begin{aligned}
S_{R-2}^O = & -\frac{1}{4\pi} \int d^2\sigma V^{-1} \text{Tr}\{ (\mathcal{D}_\# U_L^{-1})(\mathcal{D}_= U_L) + \lambda_=^\# (U_L^{-1} D_\# U_L)^2 \\
& - \int_0^1 dy (\tilde{U}_L^{-1} \frac{d}{dy} \tilde{U}_L) [(\mathcal{D}_\# \tilde{U}_L^{-1})(\mathcal{D}_= \tilde{U}_L) - (\mathcal{D}_= \tilde{U}_L^{-1})(\mathcal{D}_\# \tilde{U}_L)] \\
& - \Pi_= \stackrel{a}{=} [F_{\underline{abc}}^2 (\bar{\sigma}^{\underline{bc}})] (U_L^{-1} \mathcal{D}_\# U_L) \} \quad , \quad (4.9)
\end{aligned}$$

$$S_{R-3}^O \equiv \int d^2\sigma V^{-1} [i \psi_- (\mathcal{D}_\# + i \Pi_\# \stackrel{a}{=} A_{\underline{a}}^{\hat{a}} t_{\hat{a}}) \psi_-] \quad , \quad (4.10)$$

$$S_{R-4}^O \equiv \int d^2\sigma V^{-1} [i \psi_+ (\mathcal{D}_= + i \Pi_= \stackrel{a}{=} \tilde{A}_{\underline{a}}^{\hat{a}} t_{\hat{a}}) \psi_+] \quad . \quad (4.11)$$

In the latter two equations above, we have introduced 64 right-moving and 64 left-moving Majorana fermions (ψ_- and ψ_+ , respectively) on the worldsheet. These are the degrees of freedom that were traded by removing the lefton and righton 0-forms and 4-forms in U_R and U_L in the type-B theory.

The complete spectrum of spacetime fields includes the graviton ($g_{\underline{mn}}$), dilaton (Φ), tachyon (T), two sets of $SO(32)$ gauge fields ($A_{\underline{a}}^{\hat{a}}$ and $\tilde{A}_{\underline{a}}^{\hat{a}}$) and two Kalb-Ramond fields ($A_{\underline{mn}}^1$ and $A_{\underline{mn}}^2$). This is the spectrum of the type-O bosonic string at low orders.

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APPENDIX A: Kalb-Ramond Matter Field and 4D String σ -models

One interesting consequence of the construction of the model in ref. [1] is that a compactification of the model reveals how 4D Kalb-Ramond matter fields can be coupled to the world sheet in NSR or GS string σ -models. The idea is that the previous work utilizing Lefton-Righton Thirring Models [13, 19] can appropriately be modified to include Kalb-Ramond fields as long as the Kalb-Ramond fields do not carry any internal charges. In this brief appendix we will demonstrate how this construction is carried out within the confines of the 4D, $N = 8$ (1,0) supergravity NSR σ -model. We only pick this choice because of its simplicity. Any model with smaller values of N (or even larger values of D) can be treated by the same techniques as those we use below.

The 4D, $N = 8$ (1,0) supergravity NSR σ -model is described by using the NSR sector in (2.5) but the R-R sector is replaced by

$$\begin{aligned}
 S_{R-5} = \int d^2\sigma d\zeta^- E^{-1} i \frac{1}{2} [& (L_{\hat{=}}^{\hat{\alpha}} + \Gamma_{\hat{=}}^{\hat{\alpha}})(L_{\hat{+}}^{\hat{\alpha}} - \Lambda_{\hat{+}}^{\hat{=}}(L_{\hat{=}}^{\hat{\alpha}} + \Gamma_{\hat{=}}^{\hat{\alpha}})) + L_{\hat{+}}^{\hat{\alpha}} \Gamma_{\hat{=}}^{\hat{\alpha}} \\
 & + (R_{\hat{+}}^{\hat{I}} + 2\Gamma_{\hat{+}}^{\hat{I}})R_{\hat{=}}^{\hat{I}} - i\Lambda_{\hat{=}}^{\hat{\sharp}}(R_{\hat{+}}^{\hat{I}} + \Gamma_{\hat{+}}^{\hat{I}})\nabla_{\hat{+}}(R_{\hat{+}}^{\hat{I}} + \Gamma_{\hat{+}}^{\hat{I}}) \\
 & + 4S^{\alpha\hat{I}}R_{\hat{=}}^{\hat{I}}L_{\hat{+}}^{\hat{\alpha}} - 4\Lambda_{\hat{+}}^{\hat{=}}(M^{-1})^{\hat{I}\hat{K}}\Phi^{\hat{\alpha}\hat{I}}\Phi^{\hat{\alpha}\hat{J}}\Sigma_{\hat{=}}^{\hat{J}}\Sigma_{\hat{=}}^{\hat{K}} \\
 & - 4i\Lambda_{\hat{=}}^{\hat{\sharp}}\Phi^{\hat{\alpha}\hat{I}}L_{\hat{+}}^{\hat{\alpha}}\nabla_{\hat{+}}(\Phi^{\hat{\beta}\hat{I}}L_{\hat{+}}^{\hat{\beta}})] \quad , \tag{A.1}
 \end{aligned}$$

$$\begin{aligned}
 L_{\hat{+}}^{\hat{\alpha}} &= L_{\hat{+}}^{\hat{\alpha}} - \Lambda_{\hat{+}}^{\hat{=}}(L_{\hat{=}}^{\hat{\alpha}} + \Gamma_{\hat{=}}^{\hat{\alpha}}) \quad , \quad L_{\hat{A}}^{\hat{\alpha}} \equiv \nabla_A \varphi L^{\hat{\alpha}} \quad , \quad R_{\hat{A}}^{\hat{I}} \equiv \nabla_A \varphi R^{\hat{I}} \quad , \\
 R_{\hat{=}}^{\hat{I}} &= R_{\hat{=}}^{\hat{I}} - i[\Lambda_{\hat{=}}^{\hat{\sharp}}\nabla_{\hat{+}}(R_{\hat{+}}^{\hat{I}} + \Gamma_{\hat{+}}^{\hat{I}}) + \frac{1}{2}(\nabla_{\hat{+}}\Lambda_{\hat{=}}^{\hat{\sharp}})(R_{\hat{+}}^{\hat{I}} + \Gamma_{\hat{+}}^{\hat{I}})] \quad , \\
 \Sigma_{\hat{=}}^{\hat{I}} &= R_{\hat{=}}^{\hat{I}} - 2i[\Lambda_{\hat{=}}^{\hat{\sharp}}\nabla_{\hat{+}}(\Phi^{\hat{\beta}\hat{I}}L_{\hat{+}}^{\hat{\beta}}) + \frac{1}{2}(\nabla_{\hat{+}}\Lambda_{\hat{=}}^{\hat{\sharp}})\Phi^{\hat{\beta}\hat{I}}L_{\hat{+}}^{\hat{\beta}}] \quad , \\
 (M)^{\hat{I}\hat{J}} &= \delta^{\hat{I}\hat{J}} - 4i(\nabla_{\hat{+}}\Lambda_{\hat{+}}^{\hat{=}})\Lambda_{\hat{=}}^{\hat{\sharp}}\Phi^{\hat{\alpha}\hat{I}}\Phi^{\hat{\alpha}\hat{J}} \quad , \tag{A.2}
 \end{aligned}$$

$$\begin{aligned}
 \Gamma_{\hat{=}}^{\hat{\alpha}} &\equiv \Pi_{\hat{=}}^{\underline{a}}A_{\underline{a}}^{\hat{\alpha}}(X) \quad , \quad A_{\underline{a}}^{\hat{\alpha}} = (\tilde{A}_{\underline{a}}^{[ij]}) \quad , \\
 \Gamma_{\hat{+}}^{\hat{I}} &\equiv \Pi_{\hat{+}}^{\underline{a}}A_{\underline{a}}^{\hat{I}}(X) \quad , \quad A_{\underline{a}}^{\hat{I}} = (A_{\underline{a}}, A_{\underline{a}}^{[i'j']}, A_{\underline{a}}^{[i'j'][k'l']}) \quad . \tag{A.3}
 \end{aligned}$$

$$\Phi_{\hat{\alpha}\hat{I}} \equiv (\Phi_{[ij]}, \Phi_{[ij][i'j']}, \Phi_{[p'q'][i'j'][k'l']}, \delta_{i'[i}\delta_{j]j'}\tilde{\Phi}_{[k'l']} - \delta_{k'[i}\delta_{j]l'}\tilde{\Phi}_{[i'j']}) \quad . \tag{A.4}$$

We see in addition to the graviton, axion and dilaton which appear in the NSR sector, there also appear 28 spin-1 fields ($\tilde{A}_{\underline{a}}^{[ij]}, A_{\underline{a}}^{\hat{I}}, A_{\underline{a}}^{[i'j']}, A_{\underline{a}}^{[i'j'][k'l']}$) and 68 scalar fields ($\Phi_{[ij]}, \Phi_{[ij][i'j']}, \Phi_{[p'q'][i'j'][k'l']}, \tilde{\Phi}_{[k'l']}$) which complete the bosonic spectrum to that of 4D, $N = 8$ supergravity.

We now wish to carry out a duality transformation on the world sheet of the (1,0) string whereby some of the scalar fields are replaced by Kalb-Ramond fields. We will use the scalar field $\tilde{\Phi}_{[k'l']}$ for the purpose of illustration.

We first set to zero both $\tilde{\Phi}_{[k'l']}$ and $\phi_R^{[k'l']}$. The former operation eliminates the spacetime scalar $\tilde{\Phi}_{[k'l']}$ from among the background fields and the latter operation eliminates the modes of the string which describe this state. At this intermediate point, the theory will be inconsistent. In order to restore the consistency, new modes on the world sheet must be introduced. We can do this by introducing a new righton WZNW model precisely of the form of the R-R terms in (2.3). However, the group element that corresponds to (2.4) is here described by

$$U_{R-2} \equiv \exp\left[\frac{1}{2}\Phi_{\underline{ab}R}^{[k'l']} (\sigma^{\underline{ab}}) \otimes \mathcal{T}_{[k'l']}\right] \quad (A.5)$$

where now $\sigma^{\underline{ab}}$ denotes the right-handed Pauli matrix Lorentz generator for 4D spacetime. Also in the above expression $\mathcal{T}_{[k'l']}$ denotes a matrix representation of a $U(1)^6$ group. Note that the fact that this is an abelian group is absolutely critical. The matrices $\sigma^{\underline{ab}} \otimes \mathcal{T}_{[k'l']}$ clearly form an algebra. This would not necessarily be the case if $\mathcal{T}_{[k'l']}$ represented some non-abelian group. So the duality on the worldsheet correspond to

$$\tilde{\Phi}_{[k'l']} \rightarrow \Phi_{\underline{ab}R}^{[k'l']} \quad . \quad (A.6)$$

Finally to complete the introduction of the 4D spacetime Kalb-Ramond field, we replace the Γ_+ in (2.7) by

$$\Gamma_+ \rightarrow \Pi_+^{\underline{a}} F_{\underline{abc}}^{[k'l']} (\sigma^{\underline{bc}}) \otimes \mathcal{T}_{[k'l']} \quad . \quad (A.7)$$

We note that interestingly enough, the reduction of the type-IIA theory is expected to possess precisely six matter Kalb-Ramond fields. Thus, the final model that we have discussed (or possibly its mirror with respect to the duality transformation described above) might present an intrinsic approach to investigating heterotic type-IIA duality totally within the confines of heterotic string theory.

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